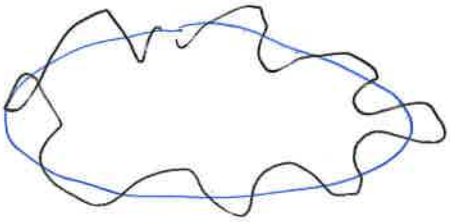
Physics : Zero modes are consequence of extremality

~~Strategy → Regulate w/~~

They are a sign that AdS_2 may not be decoupled from the environment at the quantum level.



Strategy : Regulate w $T > 0$ (small temperature)

→ Calculate $Z_{AdS_2}(T)$.

→ Take limit $T \to 0$.

This zero mode is identified w/ the Schwarzian mode in JT gravity (2d theory)

(But we will work in ~~use~~ 4d theory of gravity.) ..

§ Zero modes on AdS_2, regulated.

Reissner-Nordström BH in Einstein-Maxwell theory
$(g_{\mu\nu}, A_\mu)$

↳ Contains the essential physics [L2].

$$f(\rho) = \left(1-\frac{\rho_+}{\rho}\right)\left(1-\rho_-/\rho\right) \qquad ds^2 = -f(\rho)\,dt^2 + \frac{d\rho^2}{f(\rho)} + \rho^2 d\Omega_2^2 .$$

$$4\pi T = f'(\rho_+) = \frac{\rho_+}{\rho_+^2}\left(1-\frac{\rho_-}{\rho_+}\right) \Rightarrow 4\pi T \rho_+ = \frac{\rho_+-\rho_-}{\rho_+}.$$

$T=0 \Rightarrow \rho_+ = Q \Rightarrow (TQ) << 1$ is the relevant dimensionless parameter.

Extremal limit $TQ \to 0 \Rightarrow \frac{\rho_+-\rho_-}{\rho_+} \to 0$.

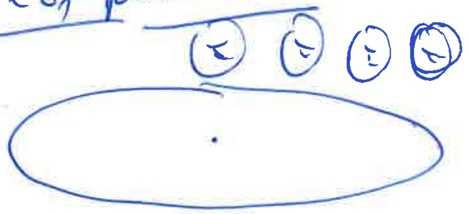
Near-horizon " $\rho - \frac{(\rho_+(T)+\rho_-(T))}{2} = 2\pi(QT)\cosh\eta$

& $t = \tau/2\pi T$

$$\Rightarrow ds^2 = Q^2\left[\left(\underbrace{(d\eta^2+\sinh^2\eta\, d\tau^2)}_{AdS_2} + \underbrace{(d\theta^2+\sin^2\theta\, d\varphi^2)}_{S^2}\right) + \delta(ds^2)\right].$$

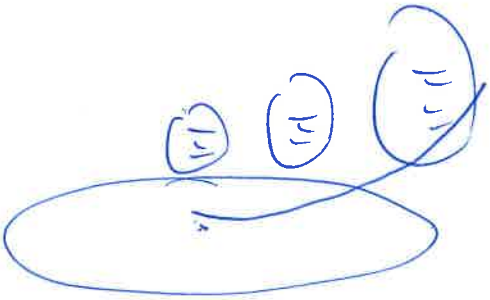
$\Rightarrow$ size of perturbation $= Q^2(QT) = Q^3 T$.

Shape of perturbation



AdS_2

$T=0$



$T>0$

$$S_{BH} = \pi Q^2 \left(1 + 4QT + O\left((QT)^2\right)\right)$$

$$= \pi Q^2 + \underbrace{4\pi Q^3 T}_{\equiv E_{gap}}$$

Exercise, but needs time

see 2209.13608

Iliesiu, S.M, Turiaci

for solution.

<u>Physics</u>: When $E \lesssim E_{gap}$, or, equivalently $T \lesssim \frac{1}{E_{gap}}$

breakdown of BH thermodynamics.

['91 Preskill - Schwartz - Shapere - Trivedi - Wilczek].

"Limitations on stat. descriptions of BHs.

not adiabatic.

<u>Modern interpretation</u>: At this scale, the nearly zero modes become strongly coupled.

Nearly zero mode : metric (Schwarzian)

→ Pure diff. (universal for all near-extremal BHs).

$$x^\mu \to x^\mu + \xi^\mu \qquad \xi = \xi_\mu dx^\mu .$$

$$\xi = \sum_{\substack{|n|\geq 2\\ n\in\mathbb{Z}}} \varepsilon_n e^{in\tau} \tanh^{|n|}\rho/2 \times \left(in\frac{(|n|+\cosh\rho)}{\sinh\rho}\partial_\rho - |n|\frac{(|n|+\cosh\rho \times \sinh^2\rho)}{\sinh^2\rho}\partial_\tau \right)$$

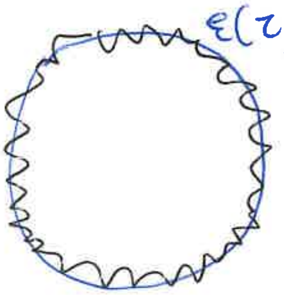
$$\underset{\rho\to\infty}{\sim} \sum_{|n|\geq 2} \varepsilon_n e^{in\tau}(in\partial_\rho - \partial_\tau).$$

$$= \varepsilon'(\tau)\partial_\rho - \varepsilon(\tau)\partial_\tau \qquad \varepsilon(\tau) = \sum_{|n|\geq 2}\varepsilon_n e^{in\tau}$$

$n = 0, \pm 1$: $SL(2)$ isometries of AdS_2.

Geometric interpretation of nearly zero modes

After coord. transf →



Fluctuations of conformal bdry of AdS_2.

Plug $\varepsilon(\tau)$ into Einstein-Maxwell action

$$\Rightarrow S[\varepsilon] = -TQ^3 \underbrace{\sum_{n\geq 2}(n^4-n^2)|\varepsilon_n|^2}_{} + \dots$$

" Schwarzian action in 2d JT gravity "

(No assumptions about 2d made here).

$$Z_{1\text{-loop}}^{z.m.} = \int [D\varepsilon]\, e^{S[\varepsilon]}$$

$$= \prod_{n\geq 2} \left(Q^3 T \right)^{-\frac{1}{2}} (\ldots)$$

quick argument: $= (Q^3 T)^{3/2}$. [See Stanford-Witten '17]

$\nearrow$ δ-function or ultralocality.

$\Rightarrow$ Vol (space of zero modes) $\underset{T\to 0}{=} (Q^3 T)^{3/2} + \ldots \to 0$!!

$$\Rightarrow \boxed{Z_{AdS_2}(Q,T) = (Q^3 T)^{3/2} \exp\left(S_0 + C_{\log} S_0 + \frac{1}{S_0} + \ldots \right)}$$

$\mathcal{N}=2$ **Supergravity** $\quad \text{Diff}(S^{1|4}) / PSU(1,1|2)$

$\mathcal{N}=4$ super-Schwarzian $\quad$ [gauge fields $g_{\mu\nu}$, A_μ, ψ_μ]

Bosonic zero modes
Fermionic "

$\Bigg\} \Rightarrow Z_{1\text{-loop}} = \int D\varepsilon_g\, D\varepsilon_\psi\, D\varepsilon_4\, e^{S_{\text{super-Schwarzian}}}$

$$= \underline{\underline{1}}\ !$$

$$\Rightarrow Z_{AdS_2\times S^2}(Q,T) = 1 \exp\left(S_0 + C_{\log} S_0 + 1/S_0 + \ldots \right)$$

Finally, ~~exactly~~ zero modes on orbifold geometries $\Rightarrow$ $\boxed{Z_{\frac{AdS_2\times S^2}{\mathbb{Z}_c}} = \frac{1}{c}}$

$\downarrow$

Precisely missing factor in orbifold localization

See
Iliesiu, S.M., Turiaci
2209.13602
13608 for details

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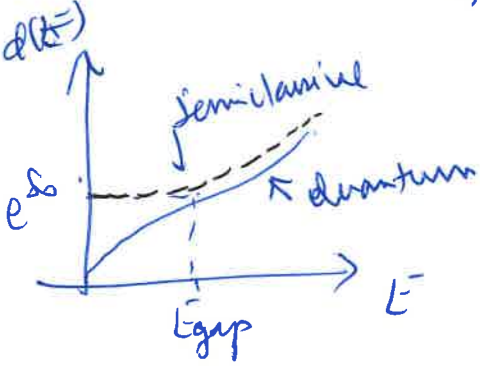
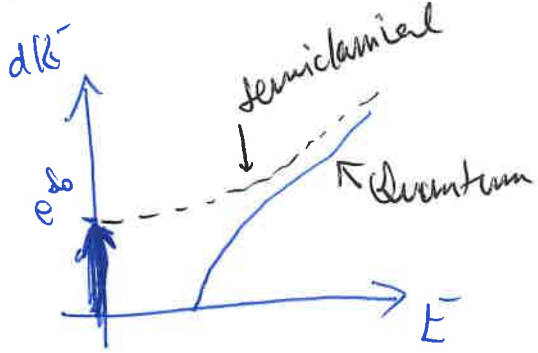
§ Interpretation of results



- Non-susy extremal
  - ⇒ No mass gap.
  - No decoupling of BH.
  - No exp. degeneracy of states at T=0.



- Susy (• extremal)
  - → Mass gap ⇒ decoupling of BH.

$$\xrightarrow{} \underset{\underline{T=0}}{d(\Delta)} = \sum_{c=1}^{\infty} \frac{1}{c^{9/2}} K_c(0)\, \tilde{I}_{7/2}(4\pi\sqrt{\Delta}/c)$$

$= \text{integer}$
$\parallel$
$\text{dim}\,\mathcal{H}_{BH} \longrightarrow$ from gravity!

Note: dim($\mathcal{H}_{BH}$) ∈ ℤ from gravity appears as the result of successive approximations of analytic effects.

cf Picture of Wheeler

§ Loose ends (index v/s degeneracy).

① Recall $B_{2n} = \frac{1}{(2n)!} \mathrm{Tr}\, (-1)^{2J_3} \underbrace{(2j_3)^{2n}}$

↳ absorbs fermion zero modes.

$\mathcal{H} = \mathcal{H}_{BH} \otimes \mathcal{H}_{out}$.

↗

~~[illegible]~~

Decoupling of susy BH imp.

$\mathcal{H}_{out}$ has exactly $2n$ fermion zeromodes ~~i.e. all broken~~ from broken susy.

$\Rightarrow 2j_3 = 2j_3^{BH} + 2j_3^{out}$.

$\Rightarrow B_{2n} = \frac{1}{(2n)!} \mathrm{Tr}_{\mathcal{H}_{BH} \otimes \mathcal{H}_{out}} (-1)^{(2j_3^{BH} + 2j_3^{out})} (2j_3^{BH} + 2j_3^{out})^{2n}$

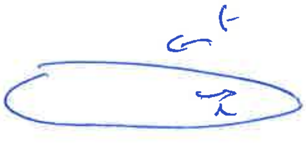
$\circledast \Rightarrow = \boxed{\frac{1}{(2n)!} \mathrm{Tr}_{\mathcal{H}_{out}} (2j_3^{out})^{2n} (-1)^{2J_3^{out}}}$.

$\quad //$

$\quad 1. \qquad \times \mathrm{Tr}_{\mathcal{H}_{BH}} (-1)^{F_{BH}}$.

$\Rightarrow$ ~~[illegible]~~ $B_{2n} = \mathrm{Tr}_{\mathcal{H}_{BH}} (-1)^{F_{BH}} \longrightarrow$ Helicity supertrace reduces to Witten index in near-horizon AdS_2.

② AdS_2



$A_t = Q r + a_0$

r fixed $\Rightarrow$ microcanonical cf ③.

microcanonical ensemble. $\Rightarrow$ All charges fixed

$\Rightarrow$ (J_3 fixed. equal for all states in ensemble).

SU(2) symmetry in grav soln ($AdS_2 \times S^2$)

$\Rightarrow \boxed{\langle J_3 \rangle = 0}$. $\Rightarrow J_3 = 0$ for each state.

$+$

$\Rightarrow$ Each state is <u>bosonic</u>

$\Rightarrow \quad \mathrm{Tr}_{\mathcal{H}_{BH}} (-1)^F = \mathrm{Tr}_{\mathcal{H}_{BH}} 1.$ [Dabholkar, Gomes, S-M., Sen '10]

BH index $=$ BH. degeneracy $= \dim \mathcal{H}_{BH}$

Also true for Schwarzian theory [Iliesiu, S.M., Turiaci '21]

③ There can be other multi-center BH bnd states in gravity w/ same charges as BH..

$\mathcal{N}=8$ theories : multi-center BH carries extra fermion z.modes

$\Rightarrow$ only single BH contributes to index. B_{14}

$\Rightarrow \quad \underline{B_{14} = \dim \mathcal{H}_{BH}}$ [Dabholkar, Gomes, S.M, Nampuri '09]

$\mathcal{N}=4$ theories : $\overset{\frac{1}{4}\text{-BPS}}{\text{BH}}$ + ~~2 BH~~ bnd state of 2 $\frac{1}{2}$-BPS BHs

contribute to B_6.

microscopic B_6 known

$\rightarrow$ Beautiful story involving wall crossing

Modular symmetry broken $\Rightarrow$ mock modular symmetry.

[Dabholkar, S.M, Zagier '12]

$\mathcal{N}=2$ theories

BHs + multi-center bnd states

B_2 not exactly known in general

Rich story waiting to be discovered!

Thanks for all the questions & comments